Formation of multiple domains and quantized vortices in two-component Bose-Einstein condensates

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This report addresses our recent study on pattern formation in two-component Bose-Einstein condensates (BECs), focusing on the dynamics of multiple-domain formation and the structure of vortex states in rotating condensates. Cross-phase modulation instability gives rise to formation of multiple domains that alternate two components, which are consistent with the experimental observation by the MIT group. Rotating two-component BECs exhibit a rich variety of vortex states such as square lattices and vortex sheets, which depend on the strength of intercomponent interaction and rotation frequency.

2 成分 Bose-Einstein 凝縮体におけるパターン形成の問題、特にマルチドメイン構造形成と量子渦状態に関する最近の我々の研究について報告する。相互位相変調の不安定性による相分離のダイナミクスを Gross-Pitaevskii 方程式の数値解析により調べ、2 成分凝縮体が交互に配列したマルチドメイン構造を形成する事を明らかにした。回転ポテンシャル中の 2 成分凝縮体は、異成分原子間相互作用と回転振動数に依存して、四角格子、渦シートなどの多彩な渦状態を形成する事が明らかとなった。

1 Introduction

Nonlinear dynamics of atomic-gas Bose-Einstein condensates (BECs) have attracted much attention recently. The nonlinearity originates from the atom-atom interaction characterized by the s-wave scattering length[1)]. This report addresses the nonlinear excitations of two-component BECs, whose dynamics are governed by the coupled Gross-Pitaevskii equations

$$i\frac{\partial \Psi_i}{\partial t} = \left[-\frac{\nabla^2}{2} + V_{\rm ext} + u_i|\Psi_i|^2 + u_{12}|\Psi_{3-i}|^2\right]\Psi_i \quad (i=1,2). \tag{1}$$

Here, Ψ_i is the condensate wave function of the i-th component, $V_{\rm ext} = (r^2 + \lambda^2 z^2)/2$ ($\lambda = \omega_z/\omega_\perp$) the harmonic potential that confines the BECs, u_i and u_{12} the atom-atom interaction between the same components and different components, respectively. All quantities in Eq. (1) are scaled by the length $b_{\rm ho} = \sqrt{\hbar/m\omega_\perp}$ (m the atomic mass and $\omega_\perp$ the radial trap frequency), time $\omega_\perp^{-1}$, and the energy $\hbar\omega_\perp$ characterizing the trapping potential. Equation (1) is formally equivalent to the incoherently coupled nonlinear Schorödinger equation familiar in nonlinear optics[2)], where the intra- and the intercomponent interactions refers to self-phase modulation and cross-phase modulation, respectively. Multicomponent BECs are produced experimentally by simultaneously trapping the atoms in different hyperfine levels or by using two or more atomic species[3)].

2 Modulation instability and multiple domain formation in two-component BECs

First, we discuss the nonlinear dynamics of multiple domain formation in two-component BECs[4], being motivated by the experiment done by Miesner *et al*[5]. They first prepared ^{23}Na Bose-condensate atoms in the $|F = 1, m = 1\rangle$ hyperfine state (component 1) and then used rf irradiation to put half of them in the $|F = 1, m = 0\rangle$ state (component 2). While the system evolved, they observed multiple domains forming with the alternating two components. Because the s-wave scattering lengths of this system are $a_1 = 2.65$nm and $a_2 = a_{12} = 2.75$nm, the system suffers from a modulation instability induced by the intercomponent repulsion (cross-phase modulation) toward phase separation. The stability analysis of miscible two-component BECs in a homogeneous system gives the unstable condition $u_1 u_2 < u_{12}^2$, where the frequencies of the out-of-phase modulation within a certain range of the wave number become purely imaginary.

To study the nonlinear dynamics of the modulation instability, we did numerical simulations of Eq. (1) by assuming axial symmetry $(x, y, z) \rightarrow (r, z)$. For the initial state, we prepared the ground state of component 1 with $N = 2 \times 10^6$, $\omega_\perp = 500$ Hz and $\lambda = 1/70$ (a highly deformed cigar-shaped potential)[5]. At $t = 0$ we transferred half of component 1 to component 2 and then let the wave functions evolve freely. Figure 1 shows the time development of $|\Psi_i|^2$ $(i = 1, 2)$ along the z-axis at $r = 0$. As soon as the simulation starts, rapidly oscillating density ripples are excited [Fig. 1(a)], which corresponds to the collective breathing mode in the tightly confined r-direction. The density wave is out-of-phase between the two components, so that the total density $|\Psi_1|^2 + |\Psi_2|^2$ keeps its initial shape. In the elongated z-direction, on the other hand, the instability grows from the edges of the elongated condensates, eventually developing into multiple domains. This is understood from the fact that the expected domain size, which may be determined by the wave number of the fastest growth mode, is smaller than the size of the z-direction but larger than that of the r-direction. These domains have a soliton-like structure, characterized by the jumps of the phase of the wave function between the domains of each component[4]. The size of one domain is about 20 μm, in good agreement with the observed domain size[5].

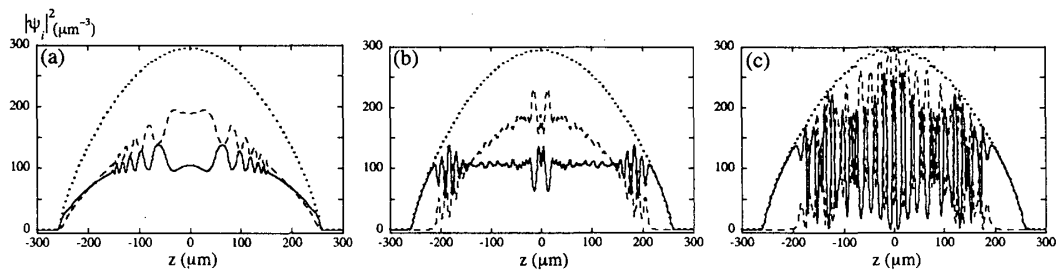

図 1: Time development of the condensate densities $|\Psi_1|^2$ (solid curve), $|\Psi_2|^2$ (dashed curve), and total density $n_T = |\Psi_1|^2 + |\Psi_2|^2$ (dotted curve) along the z axis at $r = 0$. Time of each figure is (a) $t = 40$ msec, (b) $t = 120$ msec, and (c) $t = 160$ msec.

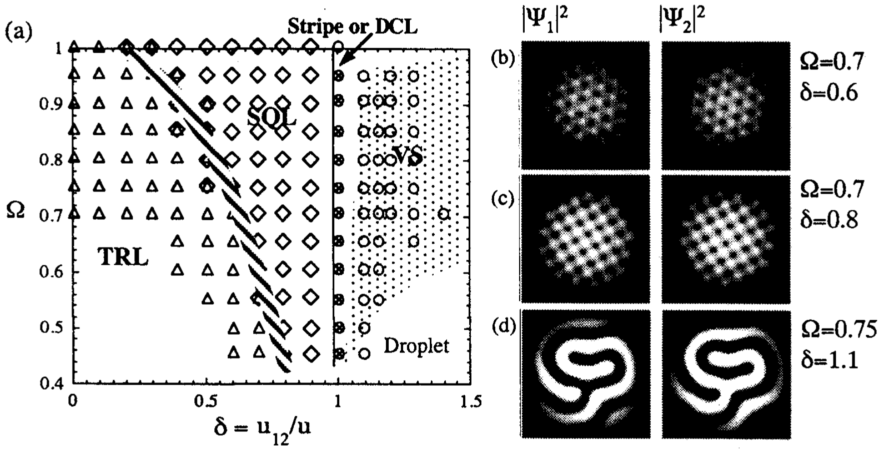

図 2: (a) Ω-δ phase diagram of the vortex states in rotating two-component BECs. The classification of the structure is made by the symbols, $\triangle$: triangular lattice (TRL), $\diamond$: square lattice (SQL), $\otimes$: stripe or double-core vortex lattice, $\circ$: vortex sheet (VS). Change from triangular to square is a continuous second order transition. The plots at $\Omega = 1$ correspond to the results of Ref. 7) based on the lowest Landau level approximation. The figures (b)-(d) show the typical equilibrium density profiles, the spatial scale of which is [-10,+10] in unit of $b_{\rm ho}$.

3 Vortex states in rotating two-component BECs

Rotating BECs form a lattice of quantized vortices which are topological defects characteristic of superfluidity. Next, we consider vortex states in rotating two-component BECs[6]. By transforming the system into a frame rotating with $\mathbf{\Omega} = \Omega\hat{\mathbf{z}}$, the centrifugal term $-\Omega L_z = -i\Omega(y\partial_x - x\partial_y)$ appears in the right hand side of Eq. (1). We are interested in the equilibrium configuration, so that the wave function is replaced as $\Psi_i(\mathbf{r},t) = \Psi_i(\mathbf{r})e^{-i\mu_i t}$ with the chemical potential μ_i. Here, the numerical calculation is done in the two-dimensional x-y space. Then, the normalization of each wave function is made as $\int dxdy|\Psi_i(x,y)|^2 = 1$ and the values of the intracomponent couplings are fixed as $u_1 = u_2 = u = 2000$ for simplicity.

We calculate the equilibrium solutions for various values of two free parameters $\delta \equiv u_{12}/u$ and Ω and summarize them in the phase diagram of Fig. 2(a). Two nonrotating components with equal mass and equal intracomponent scattering length overlap completely for $\delta < 1$ and phase-separate for $\delta > 1$. This criterion is also reflected on the structure of vortex states under rotation. The region $\delta < 1$ has triangular or square vortex lattices. For $\delta = 0$ where two components are independent, triangular vortex lattices are formed like a single-component BEC. As δ increases, the positions of vortex cores in one component gradually shift from those of the other component, changing from a triangular lattice into a square lattice [see Fig. 2(b) and (c)]; the two vortex lattices are so interlocked that each peak in the density of one component is located at each vortex core of the other. As seen in Fig. 2(a), the stable region of a square lattice depends on not only δ but also Ω. Stabilization of a square lattice is well explained by using a pseudospin representation[8], where spin-up components correspond to the density peaks of Ψ_1

at the vortex cores of Ψ_2, and vice versa for spin-down components. The case $\delta < 1$ corresponds to an antiferromagnetic interaction between the pseudospins, which makes a square lattice stable, because a triangular lattice could be frustrated. These interlaced square lattices of vortices were recently observed experimentally[9].

As the system enters a ferromagnetic phase for $\delta \geq 1$, the condensates undergo phase separation to spontaneously form domains having the same spin component. Concurrently, vortices in each component begin to overlap. Then, the density peaks of one component, at which the other-component vortices are located, merge further, resulting in the formation of "serpentine vortex sheets"; a typical example is shown in Fig. 2(d). Quantized vortices line up in sheets, and the sheets of component 1 and 2 are interwoven alternately from the center outward.

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